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Analysis of diffraction efficiency of a holographic coupler with respect to angular divergence

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Abstract : We present a method to optimize the Coupling efficiency between two fibers using a Holographic Coupler taking diffraction effect into account. For this we obtain expressions for field distributions at the end facet of a fiber. The results obtained by using the expression is found to be in good agreement with the finite element method in literature, the small deviation is attributed to the negligence of field in the cladding. We then use the Kogelnik theory to obtain useful formulae suitable for diffracted optical elements. The diffraction efficiency expressions are then used to predict the different parameters of Holograting so as to get maximum diffraction efficiency in a direction where the field distribution from the end facet is minimum thereby increasing the coupling efficiency.

Keywords : Coupling efficiency, holographic grating, diffraction efficiency.

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1. Introduction

Over recent decades numerous publications have appeared on Holographic Couplers or Holograting both in theory as well as in experiment [1–9]. Though there are several approaches like Rigorous Coupled wave analysis [3] and Beam propagation method appeared [2] in literature, the Kogelnik's theory [1] still remains the most commonly used approach for volume grating modeling. Similarly the plane wave expansion method and finite element analysis [10] are two methods used to analyze radiation field from normally cut fibers. Recently aperture antenna approach [11] has been shown to be a convenient method for analyzing the field pattern, both for normal and tapered end fibers. The goal of this work is to reduce the Kogelnik's theory and the theory of aperture antennas to practical formulae for modeling and design of diffraction optical elements (Holographic grating) of better efficiency for use in optical communication.

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2. Modal analysis for nearfield radiation pattern

In order to evaluate the radiation field of a fiber, we consider aperture antenna analysis [11]. The field of a monochromatic wave at a point defined by position coordinate *r* can be written as a superposition of plane waves and for a source free region, it can be obtained in cylindrical coordinates (ρ, υ, Φ) [7] as

$$E(\nu,\Phi) \approx jk_0.\frac{e^{-jk_0 r}}{2\pi r}\left[\cos\nu\sum_{nm} A^{nm}(\nu,\Phi)\right] \tag{1}$$

where the indices *nm* are those corresponding to LP_{nm} modes and ν is the angle between the axis and line joining between the point of observation to a point at the end faces of the fiber on the axis. κ_0 is free space wave number and A^{nm} (ν, Φ) is the modal amplitude and is given by :

$$A^{nm}(\nu,\Phi) = \cos n\phi\int_0^a j^n.\,2\pi.\,J_n(n_3 k_0\rho\sin\nu).J_n\left(\rho.\frac{u_{nm}}{a}\right)\rho.d\rho\,. \tag{2}$$

Here *J* is the Bessel function, $u_{nm} = \sqrt{n_1^2\kappa_0^2 - \beta_{nm}^2}$,

n_1 is refractive index of core, β_{nm} is propagation constant, *a* is the radius of Core of the fiber.

The near field in eq. (1) can be written as :

$$E(\nu,\Phi) \approx \frac{k_0^2}{2\pi}\left[\cos\nu\sum_{nm} A^{nm}(\nu,\Phi)\right]. \tag{3}$$

3. Diffraction of monochromatic waves on a transmission Holographic grating

From Kogelnik's theory a solution of the scalar wave equation for transmitting volume Bragg grating (VBG) gives the following formula for diffraction efficiency (DE) [1].

$$\eta = \frac{\sin^2\left\{\left(\xi^2+\phi^2\right)^{1/2}\right\}}{1+\dfrac{\xi^2}{\phi^2}} \tag{4}$$

where in eq. (4), ξ is the dephasing parameter which describes the deviation from the Bragg condition by detuning from either Bragg's angle θ_m^* or central wavelength λ_0 and can be written as,

$$\xi = \frac{\pi f t}{\cos(\phi-\theta_m^*) - (f\lambda_0/n_{a\nu})\cos\phi}\left(\Delta\theta_m\sin\theta_m^* - \frac{f}{2n_{a\nu}}\Delta\lambda\right) \tag{5}$$

where f is grating spatial frequency.

ϕ is the phase incursion in Bragg condition and can be written as

$$\phi = \frac{\pi t \delta n}{\lambda_0 F_\phi} \tag{6}$$

where t is grating thickness, δn is the refractive index modulation and F_ϕ, the inclination factor which is related to ϕ and ϕ_m^* as :

$$F_\phi = \left[-\cos(\phi - \theta_m^*)\cos(\phi + \theta_m^*)\right]^{1/2}. \tag{7}$$

Dephasing parameter ξ takes a small angular deviation $\Delta\theta_m$ from an incident Bragg angle θ_m^* and small deviation $\Delta\lambda$ from central wavelength λ_0.

For normal transmitting grating ($\phi = \pi/2$), the eq. (5) can be simplified as :

$$\xi_{\pi/2} = -\pi f t\left(\Delta\theta_m - \frac{f}{2n_{a\nu}F_{\pi/2}}\Delta\lambda\right). \tag{8}$$

The angular selectivity of a VBG for a resonant wavelength λ_0 could be determined by substituting eqs. (5) and (6) into eq. (4) at $\Delta\lambda = 0$. A simplified formula for the dependence of diffraction efficiency on detuning from the Bragg angle can be obtained as :

$$\eta(\Delta\theta_m) = \frac{\sin^2\left\{\pi t\left[(\delta n/\lambda_0 F_{\pi/2})^2 + (f\Delta\theta_m)^2\right]^{1/2}\right\}}{1 + \left(\lambda_0 f F_{\pi/2}\Delta\theta_m/\delta n\right)^2}. \tag{9}$$

4. Modeling

In Coupling Power from F_1 to F_2 by using holographic technique, the Holograting is placed in between the two fibers. The principle of coupling is the recording and reconstruction of radiation field at the end facet of fiber F_1 to F_2, where the diffraction effect is not considered. However, this cannot be ignored. Holographic principle of coupling found in literature is based on transmission of signal through the hologram by ignoring the diffraction effect. Infact, diffraction effect can only be neglected when incident light wavelength is very small compared to the dimension of the obstacle. As in the Holographic coupler we are interested, obstacle dimension is very small (Obstacle dimension is related to Λ), diffraction effect can not be ignored. We investigate the efficiency of coupling by introducing this diffraction effect. For this the radiation field from the end face of the fiber [1] is matched with diffracted field coming out of the Holographic coupler. Further we assume a single wavelength signal

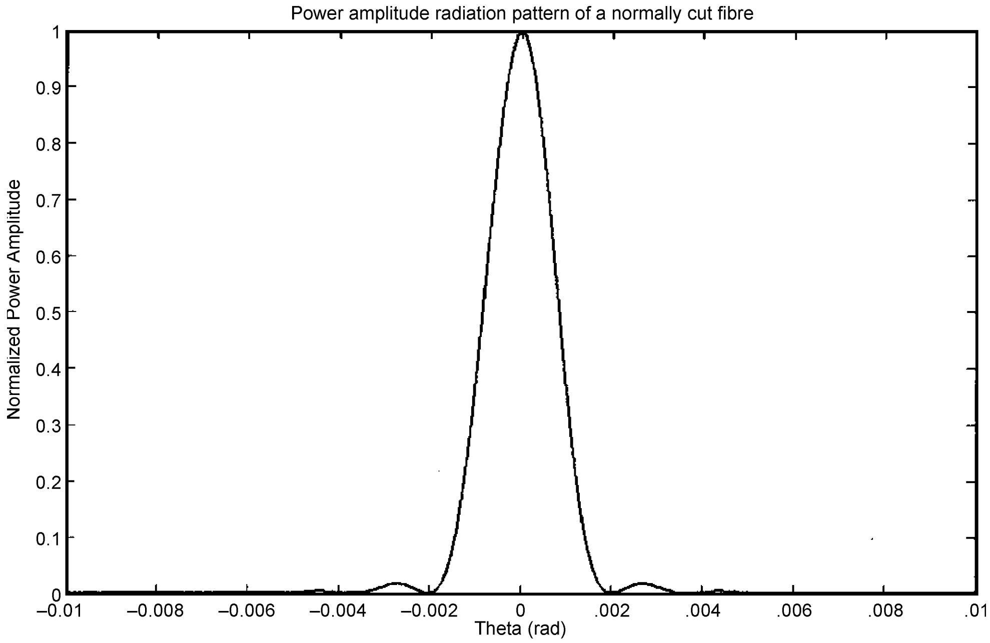

Figure 1. Field distribution from the end facet of the fiber (n_1 = 1 .501, n_2 = 1.5, n_3 = 1, $a = 20 \times 10^{-5}$ m, $\lambda_0 = 6.6 \times 10^{-7}$ m, λ = 0 radian).

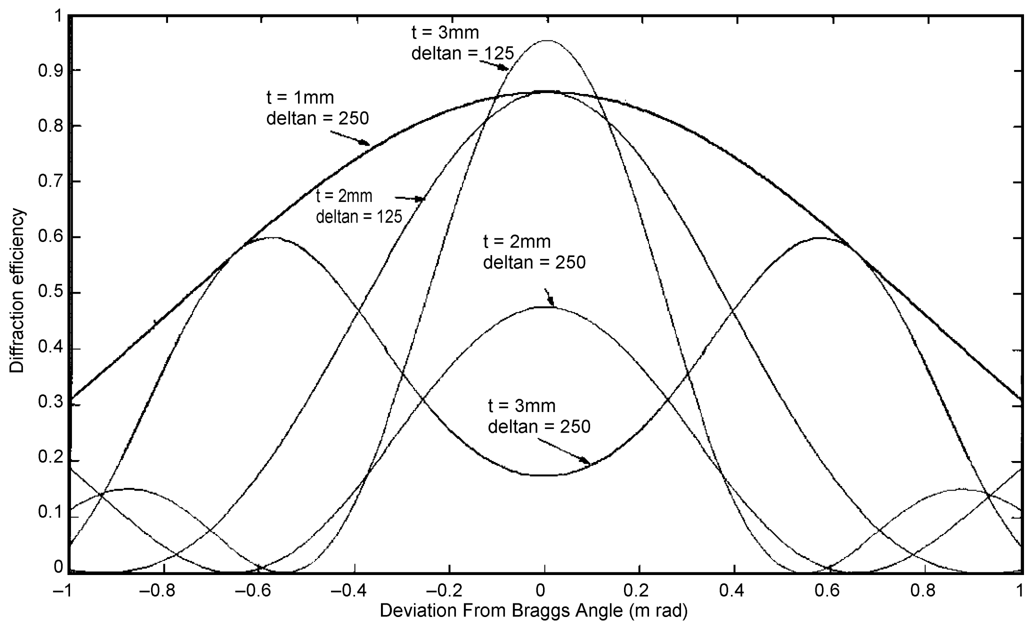

Figure 2. Dependence of diffraction efficiency on deviation from Bragg angle and wavelength with grating thickness in millimeters (t = 1 mm, $n_{a\nu}$ = 1.4867, $\lambda_0 = 6.6 \times 10^{-7}$ m).

illuminating the coupler which emerges from the fiber. The schematic diagram for this is depicted in Figure 3. Referring to Figure 3 the angle θ_i can be related to θ_m, the Bragg's angle as :

$$\theta_i = \sin^{-1}\left\{n_{a\nu}\sin\left(\theta_m + \tan^{-1}\frac{1}{ft}\right)\right\} \tag{10}$$

where $n_{a\nu}$ is the average refractive index of the Holographic Grating.

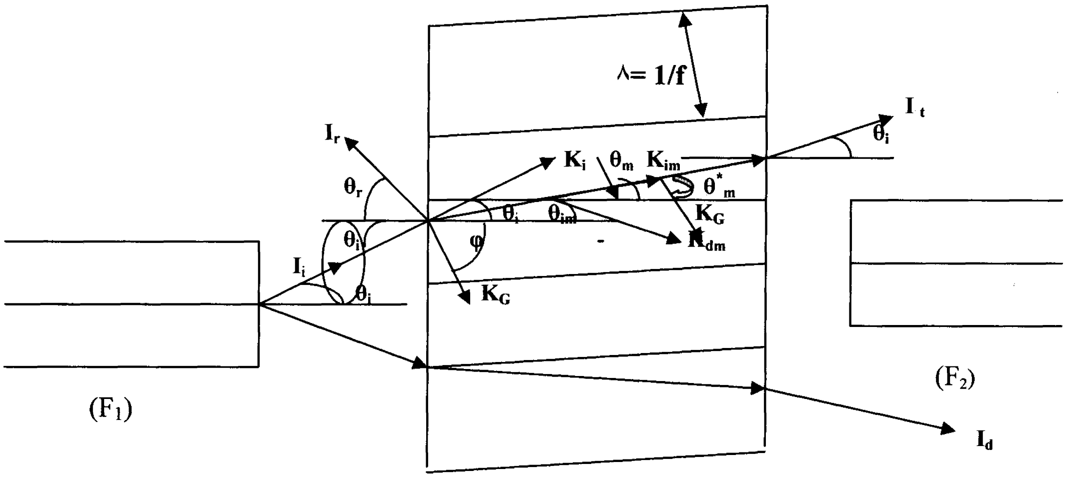

Figure 3. Propagation of optical rays through a Volume Bragg grating (I_i represents the incident beam, I_r reflected beams and I_d the diffracted beams; K_r, K_{im}, K_{dm}, are the wave vectors of incident beams in the medium; K_G grating vector; φ, the grating inclination; θ_i, θ_{im}, are angles of incidence, angle of incidence in medium θ_m Bragg angle; and θ^*_m incident Bragg angle).

5. Analysis

To investigate the validity of eq. (1), we keep the point of observation at a distance 1 mm ahead of the fiber. Our results using eq. (1) is shown in Figure 1 and is compared with that of Finite element method [8] approach. Normalized power amplitudes in our case at 2 milli radian and 10 milli radian are found as 0.0937 and 0.0092 where deviation of 0.06 and 0.007 occur in comparison to Finite element approach. This small deviation which arises here may be attributed to the negligence of radiation field beyond the core diameter which decays exponentially.

Further in Figure 2 simulation results are obtained by using eq. (8) for diffraction efficiency with the variation of Bragg's angle for different thickness and refractive index modulation. Diffraction efficiency on either side of the peak value shows less variation for t = 1 mm and refractive index modulation 250.

As the field at the end facet [1] [Figure (4)] extends in between –1 milli rad to +1 milli rad, diffraction field output from Holographic grating is to be also adjusted in the same range.

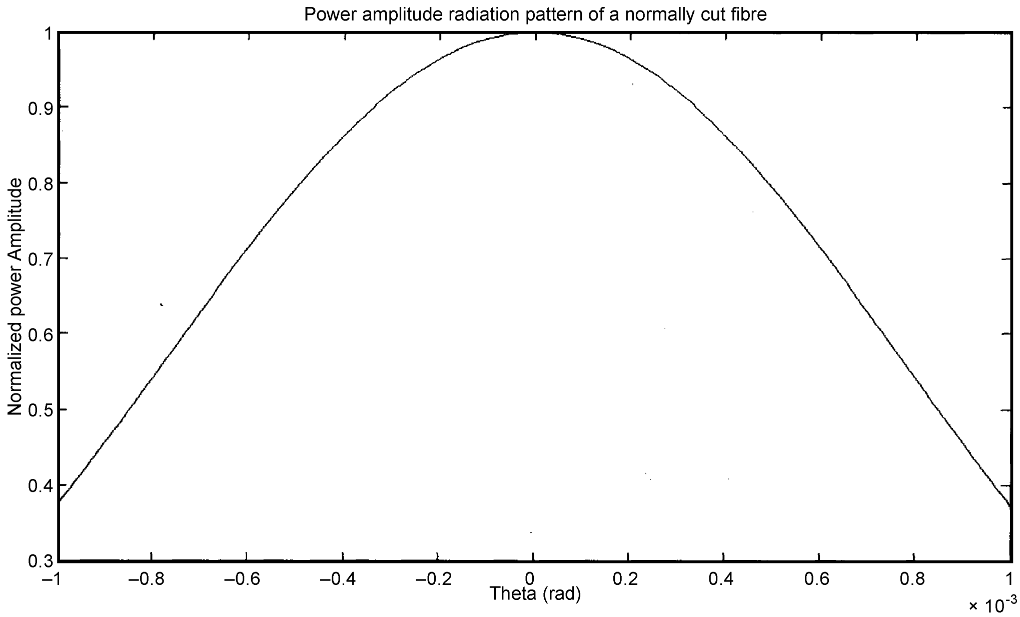

Figure 4. Field distribution from the end facet of the fiber (n_1 = 1 .501, n_2 = 1.5, n_3 = 1, $a = 20 \times 10^{-5}$ m, $\lambda_0 = 6.6 \times 10^{-7}$ m, ϕ = 0 radian).

The variation of diffraction efficiency through the Holographic Coupler with angle of incidence is found by using eqs. (8) and (9) and shown in Figures 5 and 6. The

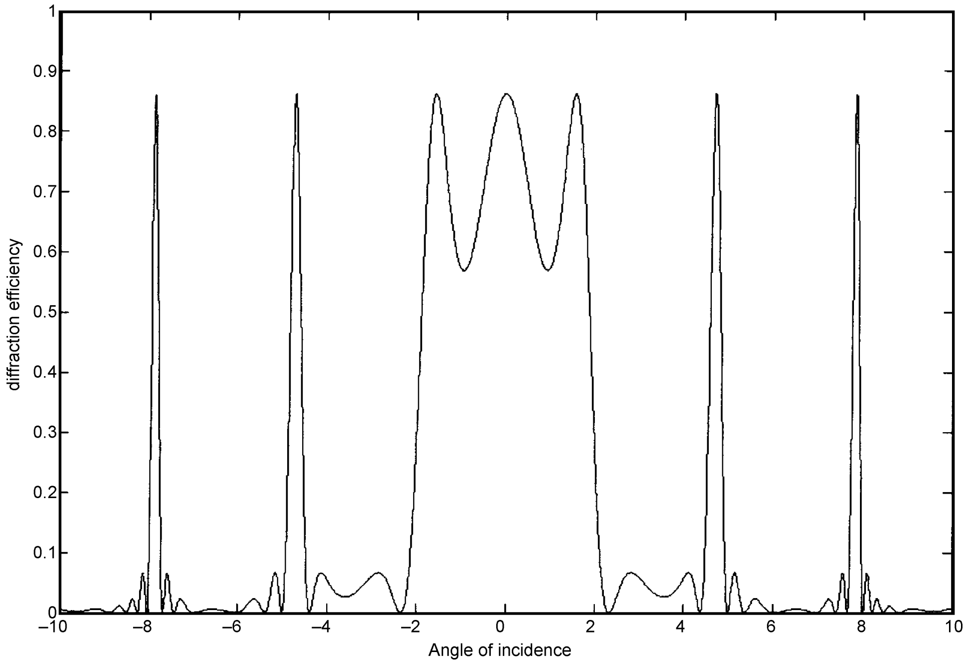

Figure 5. Dependence of diffraction efficiency on deviation from Bragg angle and wavelength with grating thickness in millimeters ($\delta n = 25 \times 10^{-6}$ mm, t_1 = 1 mm, n_{av} = 1.4867, $\lambda_0 = 6.6 \times 10^{-7}$ m).

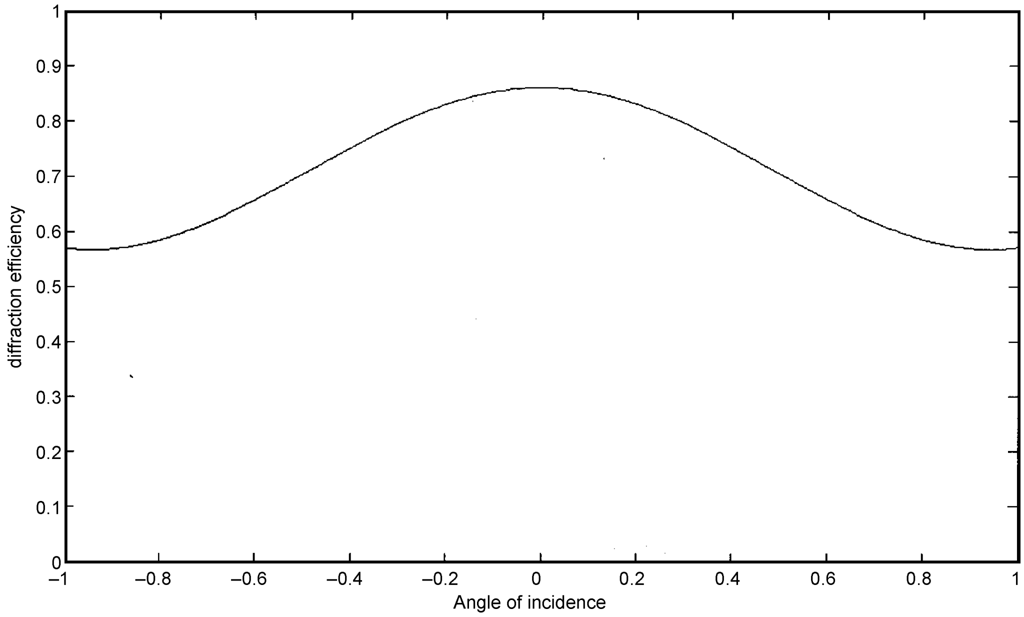

Figure 6. Dependence of diffraction efficiency on deviation from Bragg angle and wavelength with grating thickness in millimeters (t = 1 mm, $n_{a\nu}$ = 1.4867, λ_0 = 6.6 × 10^{-7} m).

radiation field from the fiber is shown only in the range –1 to +1 mill radian. The field from the fiber becomes very small at ±1.6 milli radian. The grating parameters thus are adjusted to get maximum diffraction efficiency in this direction. For a Holograting of Δn = 250 × 10^{-6} mm, t_1 = 0.001, $n_{a\nu}$ = 1.4867 the diffraction efficiency at ±1.57 mrad is 0.8619 and hence will have maximum overall Coupling efficiency. The radiation field after diffraction from fiber will match with the radiation field after diffraction from the Holographic coupler for Δn = 250 × 10^{-6} mm, t_1 = 1 mm, $n_{a\nu}$ = 1.4867. This specification Holographic Coupler will be suitable for coupling power between two fibers to give maximum coupling efficiency.

6. Conclusion

We present a method to optimize the Coupling efficiency between two fibers using a Holographic Coupler taking diffraction effect into account. For this we obtain expressions for field distributions at the end facet of a fiber. The results obtained by using the expression is found to be in good agreement with the Finite element method in literature, the small deviation is ascribed to the negligence of field in the core. Kogelnik theory is then reduced to some useful formulae suitable for diffracted optical elements. The diffraction efficiency expressions are then used to predict the different parameters of Holocoupler so as to get maximum diffraction efficiency in a direction where the field distribution from the end facet is minimum thereby increasing the coupling efficiency.

In our case it is found that for wavelength λ_0 = 6.6 × 10^{-7} m a Holocoupler of specification Δn = 250 × 10^{-6} mm, t_1 = 1 mm, $n_{a\nu}$ = 1.4867 is suitable for coupling power between two fibers having n_1 = 1.501, n_2 = 1.500, n_3 = 1, a = 20 × 10^{-5} m. This formulation can be also extended with respect to spectral width of incident beam.

Our formulation of course is applicable only for Holocoupler with sinusoidal modulation of refractive index. We believe that incorporating a non-sinusoidal refractive index modulation would give more accurate results and can be considered as future work. For this, expression for diffraction efficiency must change in accordance with the modulation index.

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